AAA 81-0299R

Gust Model Based on the Bivariate Gamma Probability Distribution

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The conventional approach to gust modeling for vertically rising vehicles involves the application of a digital filter to a sample of Jimsphere wind profiles, defining the resulting residual amplitudes as gusts, and developing statistical summaries or power spectra of the gusts. In this paper a new approach models the largest gust amplitude and gust length using the properties of the bivariate gamma distribution. The gust amplitude and gust length are strongly dependent on the filter function. Gust amplitude increases with altitude and is larger in winter than in summer.

Nomenclature

A = parameter defined by Eq. (12)
a = parameter defined under Eq. (7)
$H\{a,X\}$ = incomplete gamma function as defined by Eq. (7)
H_0 = reference altitude
H_1,H_2 = altitudes of first and second zero crossings associated with a gust
$I_{\gamma-1}\{\ \}$ = modified Bessel function of the first kind of order $\gamma-1$
j,k = indices of series approximations defined by Eqs. (6) and (7)
$L_{u'},L_{v'}$ = gust lengths associated with u and v component gusts, respectively
Q_∞ = dynamic pressure
u',v' = orthogonal gust components defined in the meteorological coordinate system in which u is the west to east (zonal) component and v is the south to north (meridional) component
x,y = bivariate gamma distributed variables
x^* = specified value of x in conditional distribution of y defined by Eq. (4)
$\bar{x}$ = sample mean of x
x_1,y_1 = integration limits defined by Eqs. (3) and (5), respectively
α = angle of attack
β^* = angle of sideslip
β = scale parameter of a gamma distribution
$\hat{\beta}$ = sample estimate of β defined by Eq. (9)
β_x,β_y = scale parameters of gamma distributed variables x and y, respectively
γ = shape parameter of gamma distribution
$\hat{\gamma}$ = sample estimate of γ defined by Eq. (10) or Eqs. (11-13b)
γ_x,γ_y = shape parameters of gamma distributed variables x and y, respectively
$\Gamma(x)$ = gamma function
λ_c = filter cutoff wavelength
ρ = correlation coefficient of variables x and y
$\hat{\sigma}$ = sample standard deviation

Presented as Paper 81-0299 at the AIAA 19th Aerospace Sciences Meeting, St. Louis, Mo., Jan. 12-15, 1981; submitted Feb. 27, 1981; revision received June 16, 1981. This paper has been declared a work of the U.S. Government and therefore is in the public domain.

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Introduction

LARGE structural loads recorded during passage of launch vehicles through patches of wind perturbations are often associated with a discrete gust that stands out above the general disturbance level.[1] To ensure a margin of safety, launch vehicle designers and launch controllers concerned with control system response and vehicle handling qualities require information about the amplitude and length scale of discrete gusts and how they vary with altitude and season. Previous models describe discrete gust as an invariant scalar quantity with an empirically derived quasi-square-wave shape and a variable wavelength.[2] Other studies based on analysis of detailed Jimsphere wind profiles between the surface and 16 km have concluded that gust amplitudes increase with altitude above 9 km.[3,4] Analytical concepts used in the development of a new gust model are described in this paper. The new model treats gust as a vector quantity that can be variable with altitude and season. The model is based on a mathematical probability concept which permits the specification of the joint relationship between gust component magnitude and length at any selected probability level. Parameters of the model are evaluated from a large sample of gust data that have been derived from Jimsphere wind profiles.

Data

A sample of detailed wind profiles at Cape Kennedy, Fla., (KSC), measured at 25-m altitude intervals with the Jimsphere FPS-16 measurement system was used in this study. The data consist of 150 profiles per month for the months of February, April, and July.[5] The February and July data are representative of the seasonal extremes at Cape Kennedy; the April data are representative of the transition between the extremes.

Data suitable for analysis was derived from these profiles by application of high-pass Martin-Graham digital filters.[6] Four filters were used which progressively allow more long-wavelength perturbations to be passed. These high-pass profiles defined here as residual profiles contain wavelengths within the 90-420, 90-997, 90-2470, and 90-6000-m wavelength bands. A set of zonal wind component, u, residual profiles calculated from the Jimsphere profile of July 28, 1965 at Cape Kennedy is illustrated in Fig. 1.

Definition of Gust

The definition of gust used in this study satisfies the objective to provide data that are suitable for detailed analysis of

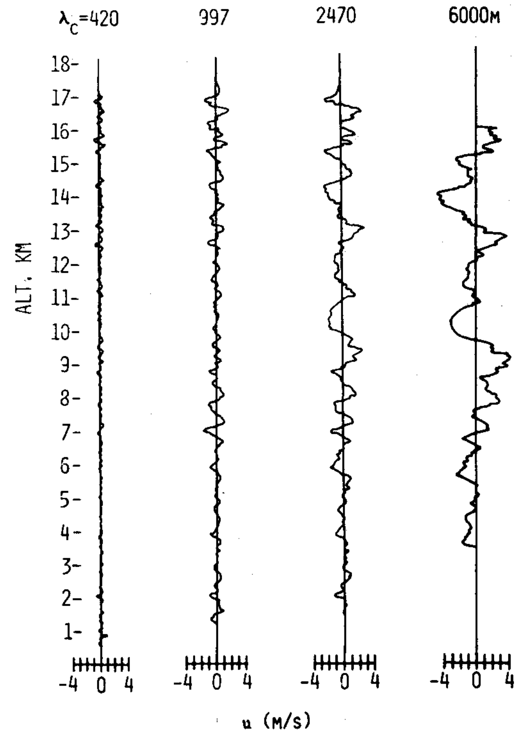

Fig. 1 Cape Kennedy residual profiles.

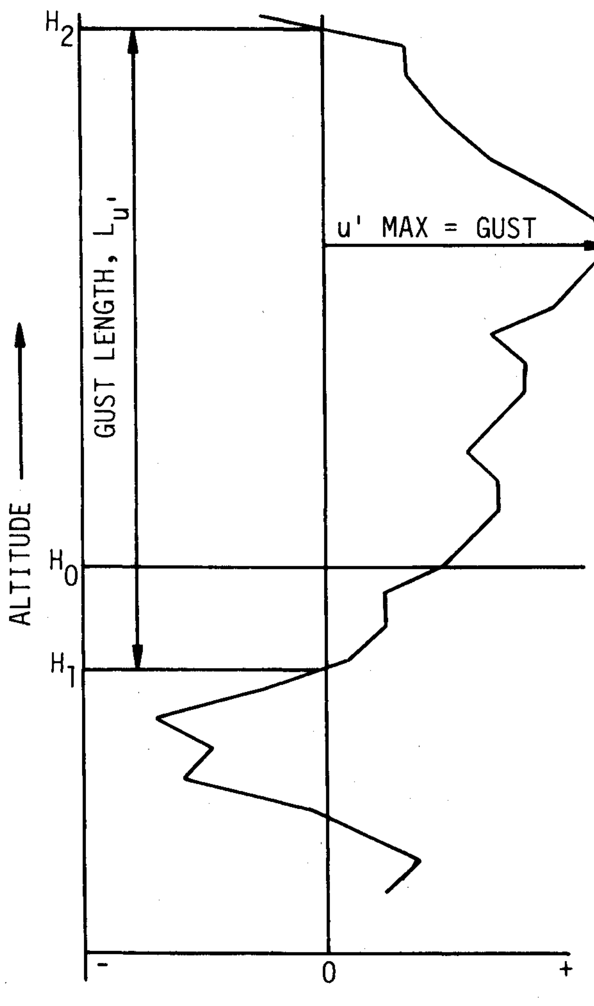

Fig. 2 Schematic definition of gust.

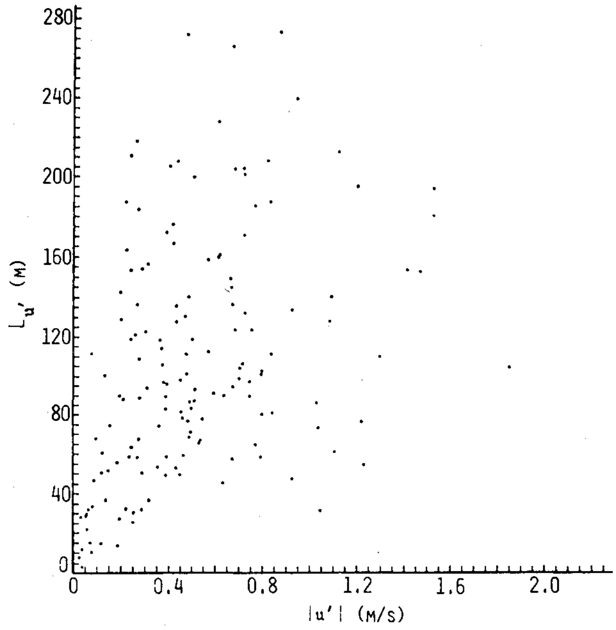

Fig. 3 Zonal component gust magnitude and associated gust length (KSC, 12 km, February, $\lambda_c = 420$ m).

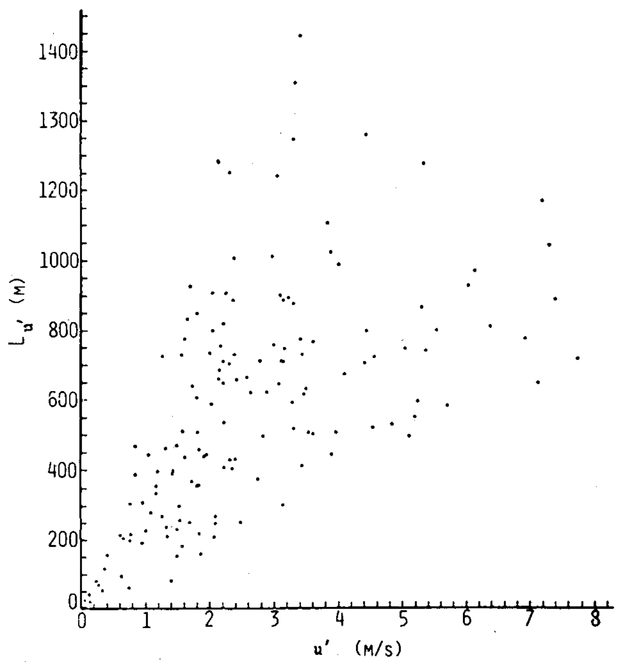

Fig. 4 Zonal component gust magnitude and associated gust length (KSC, 12 km, February, $\lambda_c = 2470$ m).

singularities or discrete perturbations that are often observed in Jimsphere wind profiles. According to the conventional approach, a gust profile would be calculated by applying a high-pass digital filter to a Jimsphere profile; all the magnitudes in the filtered profile would be defined as gusts. In this study, only the largest residual with equivalent sign to the residual at reference height, H_0, is defined as a gust; as illustrated in Fig. 2, the gust, u', occurs between successive zero crossings at H_1 and H_2; $L_{u'}$ is defined as the gust length.

Gust Data

Gust magnitudes and lengths derived from Jimsphere wind profiles according to the definition given above are illustrated in Figs. 3 and 4. The data were obtained from February profiles at a reference height of 12 km for two wavelength ranges (90-420 m, 90-2470 m). Gust magnitude increases with gust length and wavelength range. As illustrated in Fig. 3, all the gusts in the 90-420 wavelength band were less than 1.9 m/s; in contrast, the gusts in the 90-2470 wavelength band are larger by a factor of 4 (Fig. 4).

The Bivariate Gamma Distribution

The basic hypothesis of this study is that the sample of absolute component gust and associated gust length has been drawn from a bivariate gamma distribution. In this section various properties of this distribution are outlined and methods for parameter estimation are described.

Properties of the Bivariate Gamma Distribution

The bivariate gamma probability density function for equal parameters γ_x and γ_y is[7]

$$f(x,y) = \frac{(\beta_x\beta_y)^{\gamma}}{(1-\rho)\Gamma(\gamma)}\left(\frac{xy}{\rho\beta_x\beta_y}\right)^{(\gamma-1)/2} \times \exp\left(-\frac{\beta_x X+\beta_y Y}{1-\rho}\right) I_{\gamma-1}\left\{\frac{2(\rho\beta_x\beta_y XY)^{1/2}}{1-\rho}\right\} \tag{1}$$

where $\gamma=\sqrt{\gamma_x\gamma_y}$, $0\le X\le\infty$, $0\le y\le\infty$, $\gamma>0$, $\beta_x>0$, $\beta_y>0$, $0\le\rho<1$.

The marginal distribution of x is the univariate gamma distribution given by

$$f(x) = \frac{\beta_x^{\gamma_x}}{\Gamma(\gamma_x)} x^{\gamma_x-1}\exp(-\beta_x x) \tag{2}$$

The marginal distribution of y has the same form with x replaced by y. The probability that x_1 is not exceeded is

$$\Pr\{x\le x_1\} = \int_0^{x_1} f(x)\,\mathrm{d}x \tag{3}$$

The integral is the incomplete gamma function.

The conditional gamma probability density function is given by

$$f(y\,|x=x^*) = \frac{\beta_y^{\gamma}\exp\left(\frac{-\rho\beta_x x^*}{1-p}\right) y^{(\gamma-1)/2}\exp\left(-\frac{\beta_y y}{1-\rho}\right)}{(1-\rho)(\rho x^*\beta_x\beta_y)^{(\gamma-1)/2}} \times I_{\gamma-1}\left\{\frac{2(\rho\beta_x\beta_y x^* y)^{1/2}}{1-\rho}\right\} \tag{4}$$

The probability that y_1 is not exceeded given $x=x^*$ is

$$\Pr\{y\le y_1\,|x=x^*\} = \int_0^{y_1} f(y\,|x=x^*)\mathrm{d}y \tag{5}$$

The integral on the right-hand side of Eq. (5) is evaluated by the series approximation

$$\int_0^{y_1} f(y\,|x=x^*)\mathrm{d}y = \exp\left[-\left(\frac{\rho\beta_x x^*}{1-\rho}\right)\right] \times \sum_{k=0}^{\infty}\left[\frac{[\rho\beta_x x^*/(1-\rho)]^k}{k!} H\left\{\gamma+k,\frac{\beta_y y_1}{1-\rho}\right\}\right] \tag{6}$$

where $H\{a,X\}$ is the incomplete gamma function which is evaluated according to the series approximation

$$H\{a,X\} = X^a e^{-X}\sum_{j=0}^{\infty}\frac{x^j}{\Gamma(a+j+1)} \tag{7}$$

where $a=\gamma+K$

$$X=\frac{\beta_y y_1}{1-\rho} \tag{8}$$

Parameter Estimation

The parameters of the gamma distribution are estimated from sample statistics. The scale parameter, β, is calculated from an estimate of the shape parameter, γ, according to

$$\hat{\beta}=\hat{\gamma}/\bar{x} \tag{9}$$

The parameter, γ, can be estimated according to the moments method (M)

$$\hat{\gamma}=(\bar{x}/\hat{\sigma})^2 \tag{10}$$

where $\bar{x}$ and $\hat{\sigma}$ are the mean and standard deviation of the sample.

Alternatively, an estimate of γ can be obtained from either of two maximum likelihood methods; according to Thom,[8] $\hat{\gamma}$ is given by (MLI)

$$\hat{\gamma}=\frac{1}{4A}\left(1+(1+4A/3)^{1/2}\right) \tag{11}$$

where

$$A=\ln(\bar{x})-\overline{\ln(x)} \tag{12}$$

According to Bury,[9] $\hat{\gamma}$ is calculated from a polynomial approximation (MLII) of the form:

For $0\le A\le 0.577$,

$$\hat{\gamma}=(1/A)(0.5001+0.1649A-0.544A^2) \tag{13a}$$

For $0.577<A<17$,

$$\hat{\gamma}=\frac{8.899+9.060A+0.977A^2}{A(17.80+11.97A+A^2)} \tag{13b}$$

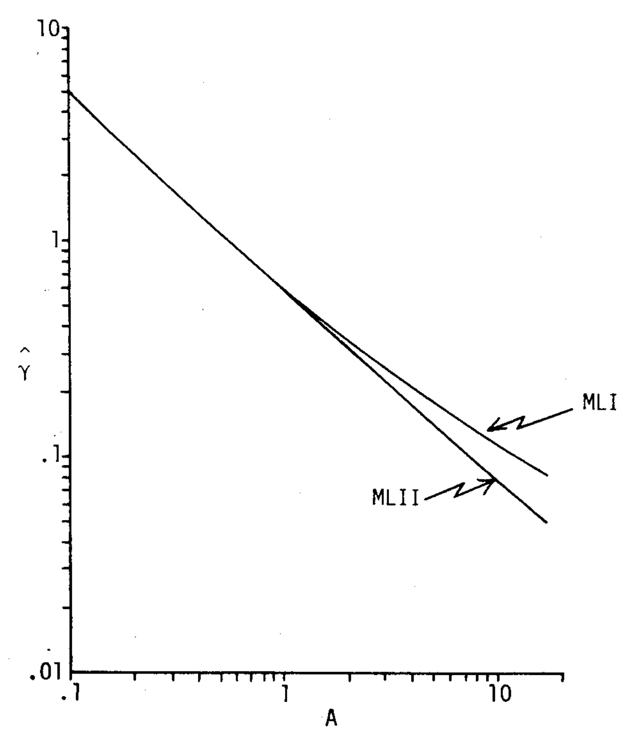

Fig. 5 Maximum likelihood estimates of γ as a function of parameter A.

Table 1 Parameter A calculated from filtered Jimsphere data during February at Cape Kennedy, Fla.

λ_c, m	Altitude, km	4	6	8	10	12	14
420	$\|u'\|$	0.2454	0.2912	0.2575	0.2030	0.2764	0.3379
	$\|v'\|$	0.2421	0.2245	0.2453	0.2285	0.2557	0.2900
	$L_{u'}$	0.1564	0.1514	0.1922	0.1308	0.2193	0.2117
	$L_{v'}$	0.1444	0.1850	0.1653	0.1802	0.2199	0.1783
997	$\|u'\|$	0.1850	0.2745	0.2541	0.1973	0.2169	0.2868
	$\|v'\|$	0.1989	0.2349	0.2297	0.2889	0.2057	0.2487
	$L_{u'}$	0.1367	0.1977	0.1818	0.2059	0.2298	0.2697
	$L_{v'}$	0.1326	0.1550	0.2061	0.2893	0.2462	0.2603
2470	$\|u'\|$	0.2009	0.2068	0.2396	0.2072	0.3120	0.2322
	$\|v'\|$	0.1875	0.2083	0.2828	0.2187	0.2601	0.2804
	$L_{u'}$	0.2145	0.1968	0.2625	0.2401	0.2807	0.2903
	$L_{v'}$	0.1929	0.1849	0.2768	0.2330	0.3759	0.3334
6000	$\|u'\|$	a	0.2970	0.2453	0.2621	0.2956	0.2040
	$\|v'\|$	a	0.2170	0.3479	0.2920	0.2943	0.2413
	$L_{u'}$	a	0.3436	0.3263	0.2851	0.3647	0.3198
	$L_{v'}$	a	0.1993	0.3370	0.2994	0.3710	0.3646

[a] Insufficient data.

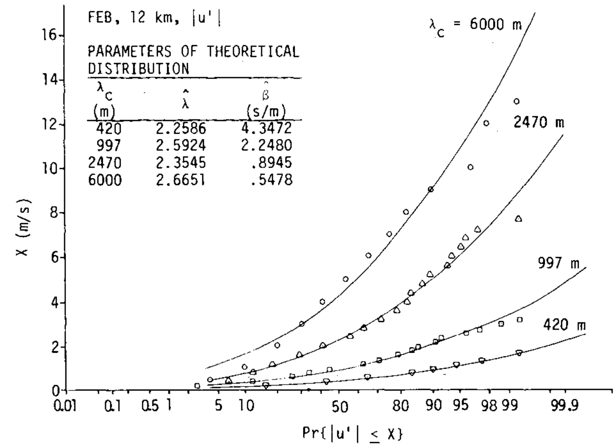

Fig. 6 Observed and theoretical (gamma) distribution of $|u'|$ as a function of λ_c (February, KSC, 12 km).

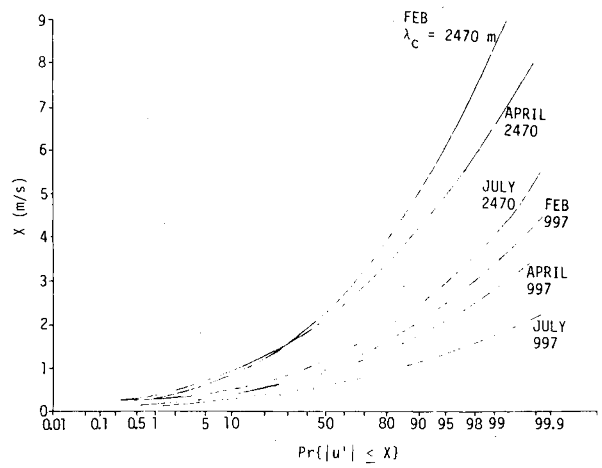

Fig. 7 Theoretical distribution of $|u'|$ (KSC, 12 km).

As illustrated in Fig. 5, there is no significant difference between the two ML methods for $A<1$. The values of parameters calculated from February sample data listed in Table 1 are less than 0.4; therefore, either ML method would be appropriate for estimation of γ. For large sample size ($m \gg 2$) and $\gamma<4$, which characterizes this data sample, the ML method is favored over the moments method.[10]

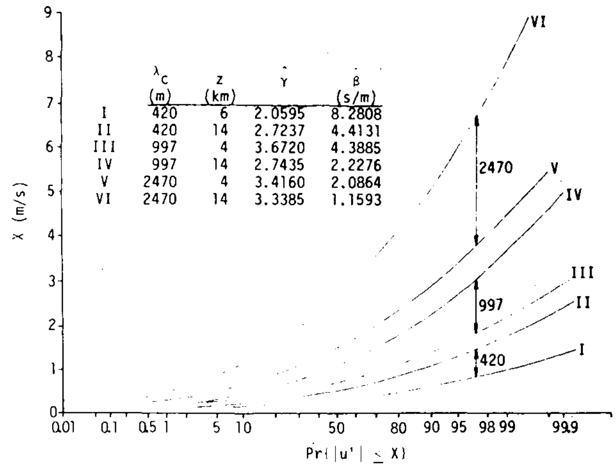

Fig. 8 Variability of theoretical distribution of $|u'|$ as a fucntion of altitude (KSC, February).

Gust Variability

In this section a few examples of observed and theoretical distributions of gust and gust length are presented. The variation of gust magnitude with filter, season, and altitude is illustrated and conditional gust magnitude given the gust length is presented for a selected reference altitude.

The large variation of gust amplitude with filter cutoff wavelength, λ_c, is illustrated in Fig. 6; no large systematic differences between the observed and theoretical distributions are indicated. Similar results obtained for gust length are not illustrated.

The variability of gust magnitude with season is illustrated by the theoretical gamma distributions in Fig. 7. The largest gust magnitudes are in February and the smallest are in July.

The variability of gust magnitude with altitude is illustrated in Fig. 8. For each filter cutoff frequency, two reference altitudes were selected to illustrate the maximum variation of the theoretical distributions in the 4-14-km altitude range. The maximum variation does not necessarily occur between 4 and 14 km.

An illustration of the conditional percentile values for the u-wind component gust amplitude given the gust length is shown in Fig. 9. Reading from this figure, the 99th percentile value of the u-component gust amplitude is 6 m/s given that the gust length is 300 m. An interesting property of the conditional gamma probability distribution function is that, as the limit of the given variable approaches zero, there

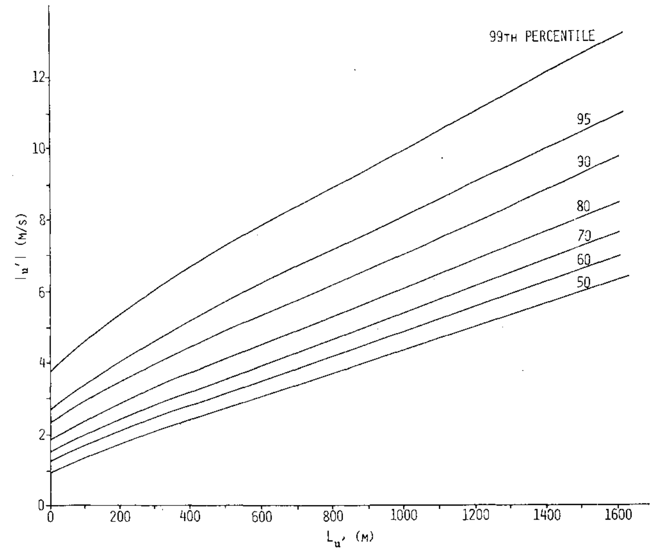

Fig. 9 Conditional percentiles of $|u'|$ given $L_{u'}$ (KSC, February, 12 km, $\lambda_c = 2470$ m).

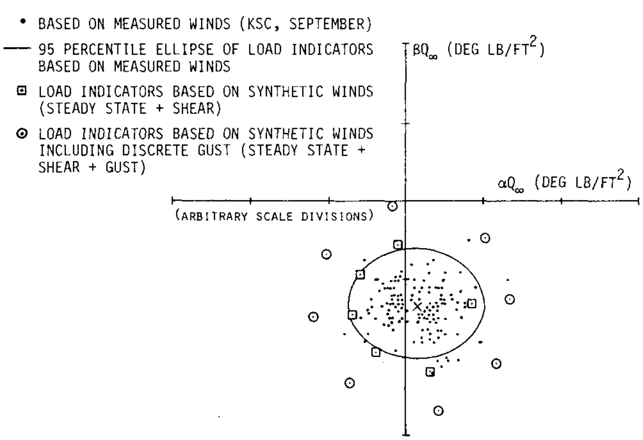

Fig. 10 Space Shuttle load indications, αQ_∞, $\beta^* Q_\infty$, from flight simulations at Mach 1.25.

remains a nonzero distribution for the other variable. For the example (Fig. 9), the inference is that, as the gust length approaches zero, the conditional 99th percentile value of the gust amplitude is 3.7 m/s.

Discussion

The widely accepted discrete wind gust model used as design criteria for aerospace ascent wind loads has a quasi-square-wave shape with an amplitude of 9 m/s and a gust length that varies from 60 to 300 m.[2] In practice, the gust amplitude is reduced by a factor of 0.85 when used with the synthetic vector wind profile model.[11] Hence, a 7.75-m/s gust is treated as a vector gust. A comparison of two aerodynamic load indicators in the pitch and yaw plane, respectively, αQ_∞ and $\beta^* Q_\infty$, with and without the gust criteria used with the synthetic vector wind model is shown in Fig. 10. Also shown in Fig. 10 is the 95% probability ellipse which contains 95% of the αQ_∞ and $\beta^* Q_\infty$ values. The trajectory is wind biased to the September vector mean wind. This comparison shows that the deterministic use of the 9-m/s gust criteria gives much larger requirements for the load indicators than that obtained for the 150 September Jimsphere wind data sample. Obviously, there is not a large wind gust (~9 m/s) near Mach 1.25 in each of the 150 September Jimsphere wind profiles. Because wind gusts cannot be predicted for flight operations, some technique must be developed to separate the contribution of wind gust to structural load from those caused by the steady-state wind and the programmed flight trajectory. Herein lies the importance of statistical modeling of wind gusts in the initial vehicle design phase which will result in a margin of safety for flight operations.

Conclusion

This paper has presented new concepts for modeling discrete wind gusts. The method involves statistical analysis of detailed wind profile measurements. The gust model for wind components is a form of the bivariate gamma probability function which yields the joint relationship between gust amplitude and gust length.

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